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HIGH RESOLUTION DYNAMICS LIMB SOUNDER

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Subject/title: Scan Mirror Angles and LOS directions

- Exact and approximate relationships between the angles turned by the scan mirror and the ILOS.
- Plots of the placement of the ILOS in the atmosphere for different orientations of the scan mirror.
- Rotation of FOV with scan.
- Change of LOS elevation with azimuth.

Changes for this issue: sign conventions made consistent with ITS. Confusing references to distortion of FOV replaced by a discussion of rotation of FOV.

Key words: Scan Mirror, Line of Sight, Field of View

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EOS

Definitions

We shall define ray directions with unit vectors $\mathbf{n}$ along the ray in the sense from the detector to the atmosphere. The components of $\mathbf{n}$ will be given with respect to the usual right-handed co-ordinate system XYZ with $+Z$ pointing towards the centre of the earth and $+X$ pointing in the direction of flight. If the unit inward mirror normal is $\mathbf{m}$, the ray vector leaving the telescope and incident upon the scan mirror is $\mathbf{t}$ and the final ray vector to the atmosphere is $\mathbf{n}$ then they are related by

$$\begin{aligned} \mathbf{n} &= \mathbf{t} - 2\,(\mathbf{t}\cdot\mathbf{m})\,\mathbf{m} \\ \mathbf{t} &= \mathbf{n} - 2\,(\mathbf{n}\cdot\mathbf{m})\,\mathbf{m} \\ \mathbf{m} &= \frac{(\mathbf{t}-\mathbf{n})}{\sqrt{2(1-\mathbf{n}\cdot\mathbf{t})}}\,. \end{aligned} \tag{1}$$

The signs of the components of $\mathbf{n}$ for atmospheric views are :

$n_x < 0$ because of rearward view

n_y can take either sign, but is negative on the sun-side for an afternoon equator crossing

$n_z > 0$ because the Earth is in the $+Z$ direction

The specific definitions of the vectors $\mathbf{m}$, $\mathbf{n}$ and $\mathbf{t}$ are as follows:

$$\mathbf{m} = \begin{pmatrix} \cos\phi_m \cos\epsilon_m \\ \sin\phi_m \cos\epsilon_m \\ -\sin\epsilon_m \end{pmatrix} \tag{2}$$

$$\mathbf{n} = \begin{pmatrix} -\cos\phi_{\mathrm{LOS}} \cos\epsilon_{\mathrm{LOS}} \\ -\sin\phi_{\mathrm{LOS}} \cos\epsilon_{\mathrm{LOS}} \\ \sin\epsilon_{\mathrm{LOS}} \end{pmatrix} \tag{3}$$

$$\mathbf{t} = \begin{pmatrix} \cos\phi_{\mathrm{FOV}} \cos(\epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}}) \\ -\sin\phi_{\mathrm{FOV}} \cos(\epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}}) \\ \sin(\epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}}) \end{pmatrix} \tag{4}$$

where $\epsilon_{\mathrm{POA}} = 25.3°$. These equations fully define the angles involved, but for clarity the following describes the definitions.

(i) The *datum mirror position* has the reflecting surface parallel with the YZ plane:

$$\mathbf{m}_{\mathrm{datum}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

(ii) The mirror is rotated from the datum mirror position to the position specified by ϕ_m and ϵ_m by rotating it through ϵ_m about the $+Y$ axis (in a right-handed sense) followed by ϕ_m about the $+Z$ axis. These are exactly the rotations induced by motions about the nominal gimbal axes (in other words, assuming perfect alignment between the actual rotation axes and the co-ordinate system, and that the elevation axis is parallel with the mirror surface) and so ϵ_m and ϕ_m are the angles measured by the elevation and azimuth encoders.

(iii) The final ray vector $\mathbf{n}$ is generated by applying to the $-X$ axis rotations of ϵ_{LOS} about the $+Y$ axis followed by ϕ_{LOS} about the $+Z$ axis.

(iv) The telescope ray vector $\mathbf{t}$ is generated either

a) by rotating the Projected Optical Axis (POA)

$$\mathbf{t}_{\mathrm{POA}} = \begin{pmatrix} \cos\epsilon_{\mathrm{POA}} \\ 0 \\ \sin\epsilon_{\mathrm{POA}} \end{pmatrix}$$

by ϵ_{FOV} about the $-Y$ axis followed by ϕ_{FOV} about the $-Z$ axis or

b) by rotating the $+X$ axis by $\epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}}$ about the $-Y$ axis followed by ϕ_{FOV} about the $-Z$ axis.

(v) The least geocentric distance R of the ray after leaving the instrument is

$$R = R_{\mathrm{EOS}} \cos\epsilon_{\mathrm{LOS}} = R_{\mathrm{EOS}} \sqrt{n_x^2 + n_y^2} \tag{5}$$

where R_{EOS} is the orbit radius (see Fig 1). If the oblateness of the Earth is neglected then the point of least geocentric distance is also the tangent point, and $R - R_E$ is the atmospheric height at that point, where R_E is the mean earth radius ($R_E = 6371$ km).

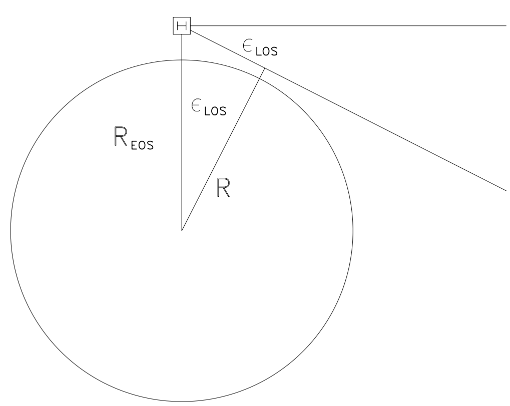

Figure 1: Limb Viewing Geometry

Four points about the above definitions should be noted:

1) Increasing ϵ implies decreasing R.
2) The subscript LOS denotes the line of sight of an arbitrary point in the instrument field defined by ϕ_{FOV} and ϵ_{FOV}; ILOS is not used here as this is defined in the ITS as the LOS of the POA.

3) The rotations about the $-Z$ and $-Y$ axes in (iv) above are defined to ensure that ϕ_{FOV} and ϵ_{FOV} increase for rays that are reflected to increasing ϕ_{LOS} and ϵ_{LOS}.
4) Because the Z axis is not orthogonal to $\mathbf{t}$, the rotation by ϕ_{FOV} about the $-Z$ axis generates a smaller angular change in $\mathbf{t}$ of roughly $\phi_{\mathrm{FOV}} \cos \epsilon_{\mathrm{POA}}$. Thus, for example, if the angular distance between two ends of a single channel FOV is 3.33 mrad, then the azimuthal distance is 3.68 mrad.

For reference we give the rotation matrix from which embodies these definitions, for a rotation of ϵ about the $+Y$ axis followed by ϕ about the $+Z$ axis:

$$\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\epsilon & 0 & \sin\epsilon \\ 0 & 1 & 0 \\ -\sin\epsilon & 0 & \cos\epsilon \end{pmatrix} = \begin{pmatrix} \cos\phi\cos\epsilon & -\sin\phi & \cos\phi\sin\epsilon \\ \sin\phi\cos\epsilon & \cos\phi & \sin\phi\sin\epsilon \\ -\sin\epsilon & 0 & \cos\epsilon \end{pmatrix}$$

By operating on the appropriate vector, and with appropriate values of ϵ and ϕ, the descriptions above can easily be shown to correspond to the actual vectors in (2) – (4).

For a spherical earth, the horizon is a line of constant ϵ_{LOS}, and the vertical through any point on the earth's surface appears as a line of constant ϕ_{LOS}. (If oblateness and the Earth's rotation are taken into account, neither of these statements is exactly true: the vertical at a point is normal to the surface, but the horizon is not a line of constant ϵ_{LOS}.)

Figure 2a shows the points of intersection of the vectors $\mathbf{n}$, $-\mathbf{m}$ and $-\mathbf{t}$ with a unit sphere, with the mirror in the datum position, and the angles ϵ_{FOV} and $\phi_{\mathrm{FOV}} = 0$. The laws of reflection state firstly that these vectors are coplanar, so that the points lie on a great circle on the sphere, and secondly that the angles between them are equal. Figure 2b shows the displacement of these points from the datum position for positive values of ϵ_{FOV}, ϕ_{FOV}, ϵ_m, ϕ_m.

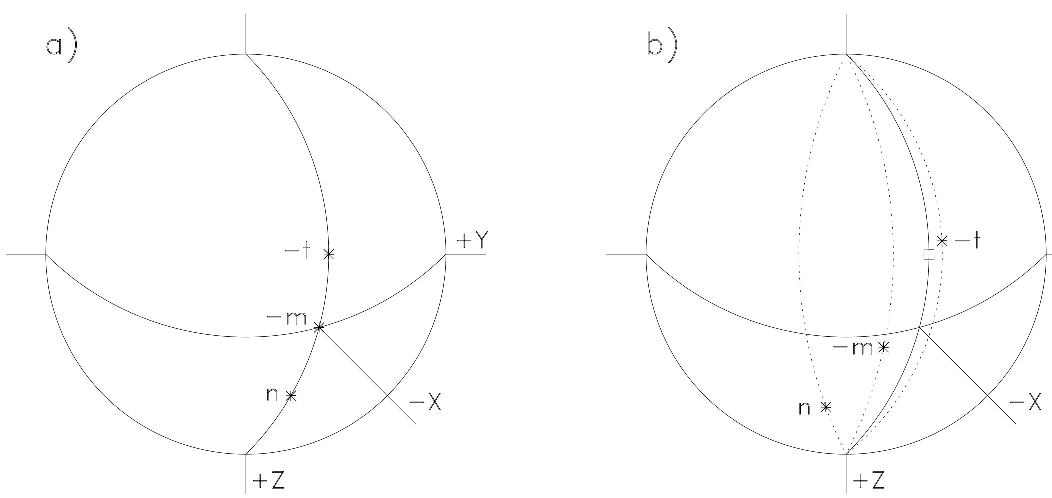

Figure 2: Reflection Geometry

Results

Substituting (2) – (4) into (1) gives equations that define ϵ_{LOS} and ϕ_{LOS}, or ϵ_{FOV} and ϕ_{FOV}, or ϵ_m and ϕ_m in terms of the other angles:

$$\begin{aligned}\sin\epsilon_{\mathrm{LOS}} &= \sin(\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}})\cos 2\epsilon_m \\ &\quad + \cos(\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}})\sin 2\epsilon_m\cos(\phi_m+\phi_{\mathrm{FOV}}) \\ \sin(\phi_{\mathrm{LOS}}+\phi_{\mathrm{FOV}})\cos\epsilon_{\mathrm{LOS}} &= \sin 2(\phi_m+\phi_{\mathrm{FOV}})\cos(\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}})\cos^2\epsilon_m \\ &\quad - \sin(\phi_m+\phi_{\mathrm{FOV}})\sin(\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}})\sin 2\epsilon_m\end{aligned} \tag{6}$$

$$\begin{aligned}\sin(\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}}) &= \sin\epsilon_{\mathrm{LOS}}\cos 2\epsilon_m \\ &\quad - \cos\epsilon_{\mathrm{LOS}}\sin 2\epsilon_m\cos(\phi_{\mathrm{LOS}}-\phi_m) \\ \sin(\phi_{\mathrm{FOV}}+\phi_{\mathrm{LOS}})\cos(\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}}) &= \sin 2(\phi_{\mathrm{LOS}}-\phi_m)\cos\epsilon_{\mathrm{LOS}}\cos^2\epsilon_m \\ &\quad + \sin(\phi_{\mathrm{LOS}}-\phi_m)\sin\epsilon_{\mathrm{LOS}}\sin 2\epsilon_m\end{aligned} \tag{7}$$

$$\begin{aligned}\sin\epsilon_m &= \frac{\sin\delta\epsilon}{D\cos\bar\phi} \\ D\sin\phi_m\cos\epsilon_m &= \sin\delta\phi\cos\delta\epsilon - \cos\delta\phi\sin\delta\epsilon\tan\bar\phi\tan\bar\epsilon\end{aligned} \tag{8}$$

where, in (8):

$$\begin{aligned}D &= \sqrt{1+\tan^2\bar\phi\sin^2\delta\epsilon/\cos^2\bar\epsilon} \\ \bar\epsilon &= (\epsilon_{\mathrm{LOS}}+\epsilon_{\mathrm{POA}}+\epsilon_{\mathrm{FOV}})/2 \\ \delta\epsilon &= (\epsilon_{\mathrm{LOS}}-\epsilon_{\mathrm{POA}}-\epsilon_{\mathrm{FOV}})/2 \\ \bar\phi &= (\phi_{\mathrm{LOS}}+\phi_{\mathrm{FOV}})/2 \\ \delta\phi &= (\phi_{\mathrm{LOS}}-\phi_{\mathrm{FOV}})/2\end{aligned}$$

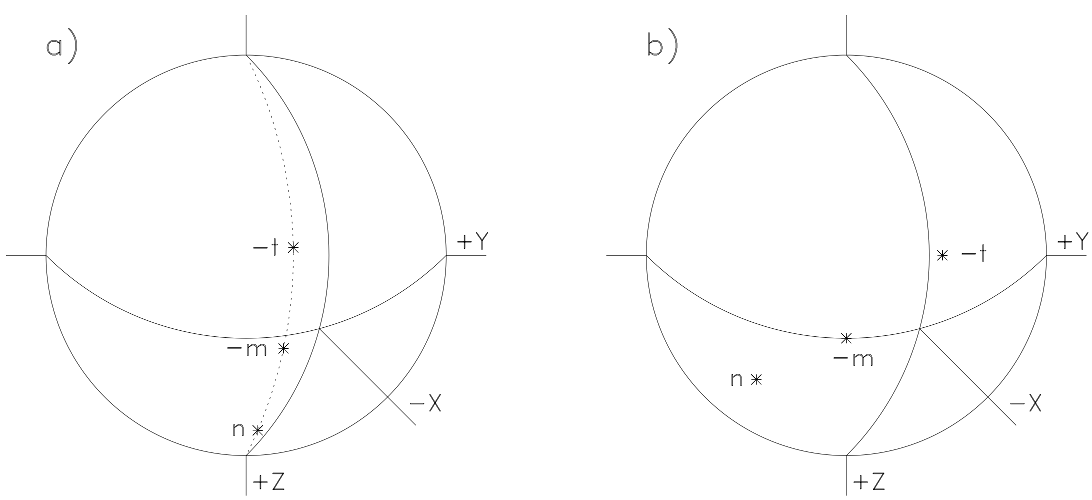

Figure 3: Special Cases

Although straightforward to solve numerically, these equations have no simple solution except in two special cases. One of these is illustrated in Figure 3a, where the rays **n** and **t** and the normal **m** lie in the same meridional plane.

$\boldsymbol{\phi_{\mathrm{m}} + \phi_{\mathrm{FOV}} = 0}$:

$$\begin{aligned} \epsilon_{\mathrm{LOS}} &= \epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}} + 2\epsilon_m \\ \phi_{\mathrm{LOS}} &= \phi_m \end{aligned}$$

The second is illustrated in Figure 3b, where the mirror normal lies in the horizontal (XY) plane.

$\boldsymbol{\epsilon_{\mathrm{m}} = 0}$:

$$\begin{aligned} \epsilon_{\mathrm{LOS}} &= \epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}} \\ \phi_{\mathrm{LOS}} &= 2\phi_m + \phi_{\mathrm{FOV}} \end{aligned}$$

These results suggest what we might term the zeroth approximation, namely that the scan mirror simply scans the field of view in elevation and azimuth without distortion.

Zeroth Approximation:

$$\begin{aligned} \epsilon_{\mathrm{LOS}} &= \epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}} + 2\epsilon_m \\ \phi_{\mathrm{LOS}} &= 2\phi_m + \phi_{\mathrm{FOV}} \end{aligned} \tag{9}$$

This is consistent with both of the above cases but is not an exact solution of (6) – (8). The fact that it is correct for $\epsilon_m = 0$ over the whole range of ϕ_m, and that the required range of ϵ_m is small suggests a first-order approximation in ϵ_m, where $\sin\epsilon_m$ is replaced by ϵ_m (in radians), $\cos\epsilon_m$ by 1, and terms quadratic in ϵ_m are neglected. (In the case of (8) the approximation is first order in $\delta\epsilon$.)

First-Order Approximation:

$$\epsilon_{\mathrm{LOS}} = \epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}} + 2\epsilon_m \cos(\phi_m + \phi_{\mathrm{FOV}}) \tag{10a}$$

$$\phi_{\mathrm{LOS}} = 2\phi_m + \phi_{\mathrm{FOV}} + 2\epsilon_m \sin(\phi_m + \phi_{\mathrm{FOV}}) \tan(\epsilon_{\mathrm{POA}} + \epsilon_{\mathrm{FOV}}) \tag{10b}$$

$$\epsilon_{\mathrm{FOV}} = \epsilon_{\mathrm{LOS}} - \epsilon_{\mathrm{POA}} - 2\epsilon_m \cos(\phi_{\mathrm{LOS}} - \phi_m) \tag{11a}$$

$$\phi_{\mathrm{FOV}} = \phi_{\mathrm{LOS}} - 2\phi_m - 2\epsilon_m \sin(\phi_{\mathrm{LOS}} - \phi_m) \tan\epsilon_{\mathrm{LOS}} \tag{11b}$$

$$2\epsilon_m = \frac{(\epsilon_{\mathrm{LOS}} - \epsilon_{\mathrm{POA}} - \epsilon_{\mathrm{FOV}})}{\cos\bar{\phi}} \tag{12a}$$

$$2\phi_m = \phi_{\mathrm{LOS}} - \phi_{\mathrm{FOV}} - 2\delta\epsilon \tan\bar{\epsilon} \tan\bar{\phi} \tag{12b}$$

The above results are consistent with John Barnett's preliminary study (dated 16th March 1990). Because of the difference in theoretical approach it is not straightforward to compare equations directly between his study and (6), but direct comparison of the numbers in his table with the results of (6) gives complete agreement, except that his numbers seem not to be accurate to all quoted decimal places at zero azimuth, probably because of the different numerical method.

In the plots we shall compare the exact results derived from (6) with the approximate results (9) and (10).

Discussion

We shall divide the discussion of these results into three parts: we shall first consider the effect of scanning on the ILOS, then the rotation of the field of view with scanning, and finally the implications for the error budget for ϕ_m and ϕ_{FOV}.

The ILOS $\mathbf{n}_{\mathrm{ILOS}}$ is defined as the reflected direction of the POA $\mathbf{t}_{\mathrm{POA}}$. The ILOS elevation and azimuth ϵ_{ILOS} and ϕ_{ILOS} can thus be found from (6), setting ϵ_{FOV} and ϕ_{FOV} to zero. Figure 4 shows the effect of scanning the mirror in ϵ_m at constant ϕ_m and ϕ_m at constant ϵ_m, plotted in terms of ϕ_{ILOS} and height $(R - R_E)$ using (5) and (6). The figure is symmetrical about $\phi_{\mathrm{ILOS}} = 0$ and so only negative values of ϕ_{ILOS} are plotted. Whereas the zeroth approximation would produce a rectangular grid, the distortion of the scan pattern away from zero azimuth or elevation is obvious.

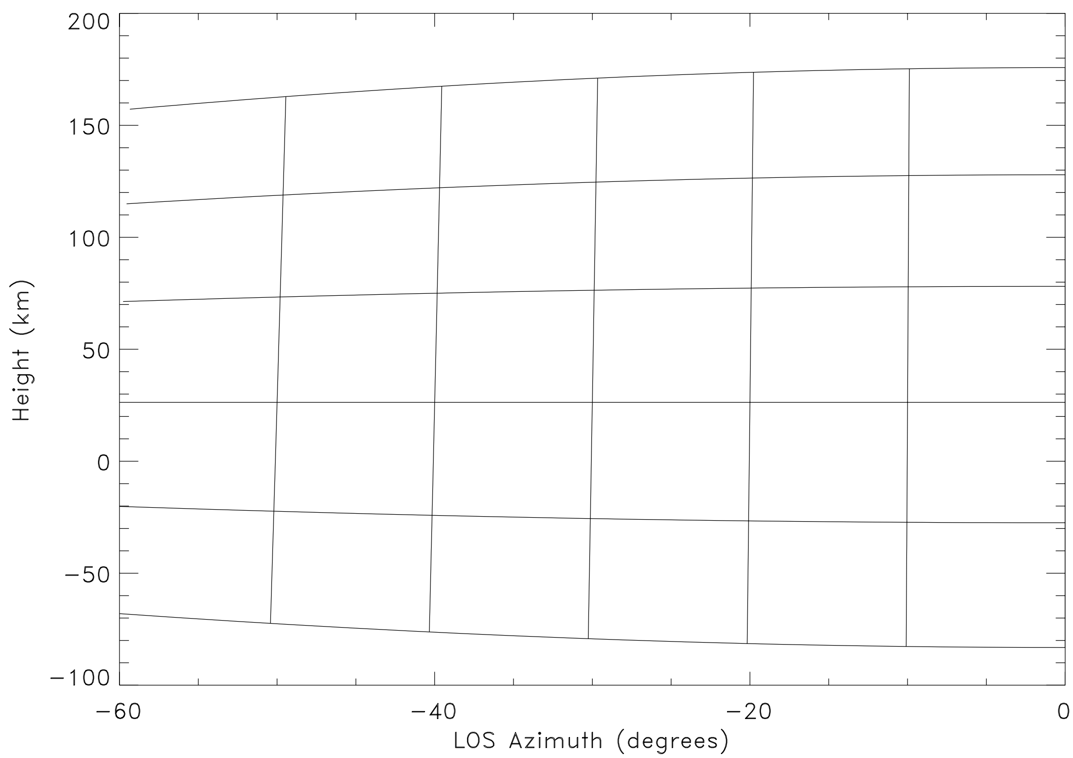

Figure 4: ILOS Scanning

The first-order approximation for ϵ_{ILOS}, ϕ_{ILOS} is found from (10):

$$\epsilon_{\mathrm{ILOS}} = \epsilon_{\mathrm{POA}} + 2\epsilon_m \cos\phi_m \tag{13a}$$

$$\phi_{\mathrm{ILOS}} = 2\phi_m + 2\epsilon_m \sin\phi_m \tan\epsilon_{\mathrm{POA}} \tag{13b}$$

The $\cos\phi_m$ factor multiplying ϵ_m in (13a) gives a contraction of the vertical scan range at non-zero azimuth, which is seen in the convergence of the lines of constant ϵ_m towards each other, and the extra term involving ϵ_m in (13b) as compared with the zeroth approximation gives a slant to the lines of constant ϕ_m. Thus at least qualitatively Figure 4 is consistent with the first-order approximation (13). The quantitative comparison is given in Figure 5, which shows the error in the zeroth approximation, scanning ϵ_m at constant ϕ_m of $-22°$. The dotted line shows the correction to the zeroth approximation given by the first-order approximation (13). It is clear that while (13) is probably not sufficiently accurate for use in retrievals, it is perfectly adequate for setting scan ranges, and error analysis.

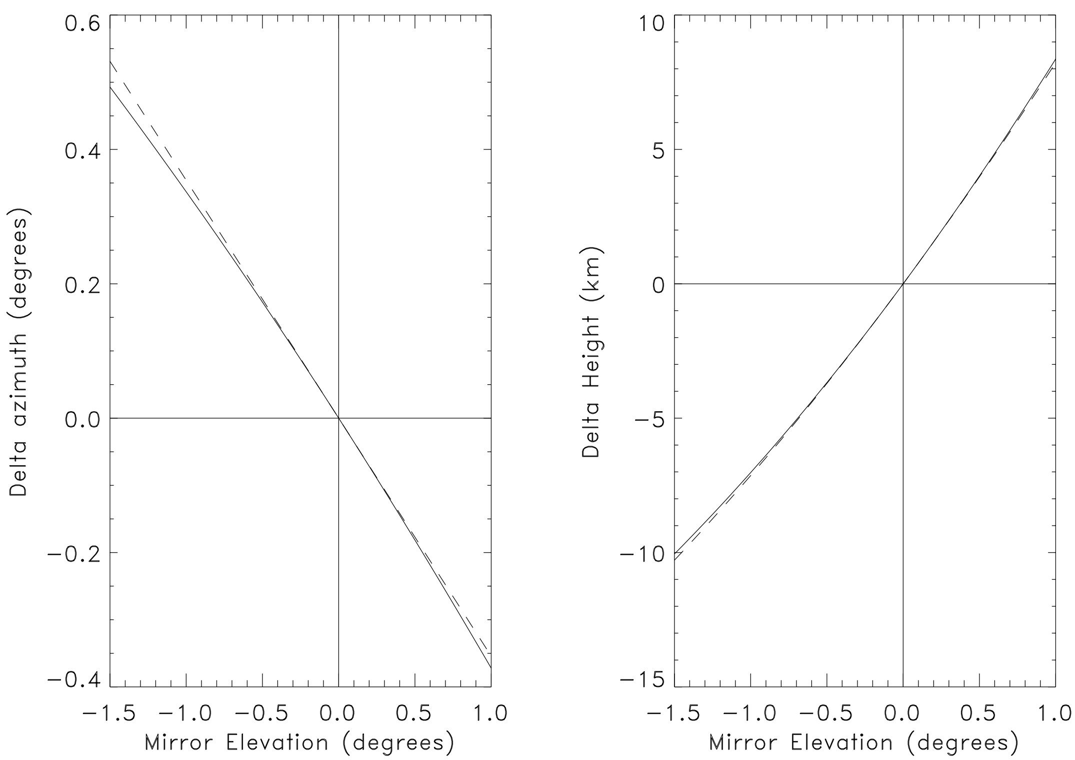

Figure 5: Error in Zeroth Approximation

To consider the rotation of the field of view we calculate the angular differences between an LOS ray and the ILOS, $\Delta\epsilon = \epsilon_{\mathrm{LOS}} - \epsilon_{\mathrm{ILOS}}$ and $\Delta\phi = \phi_{\mathrm{LOS}} - \phi_{\mathrm{ILOS}}$, in terms of the angles ϵ_{FOV} and ϕ_{FOV} corresponding to some fixed point in the FOV. Subtracting (13) from (10) gives the first-order approximation to these quantities, which we expand to first order in the small angles ϕ_{FOV} and ϵ_{FOV}. In oder to write the result as a rotation we need equal scale factors along both axes, so we use the angular separations corresponding to the azimuthal distances, defined as $\psi_{\mathrm{FOV}} = \phi_{\mathrm{FOV}} \cos\epsilon_{\mathrm{POA}}$ and $\Delta\psi = \Delta\phi\cos\epsilon_{\mathrm{LOS}}$.

$$\begin{pmatrix} \Delta\psi \\ \Delta\epsilon \end{pmatrix} = \begin{pmatrix} 1 & 2\epsilon_m \sin\phi_m \sec\epsilon_{\mathrm{POA}} \\ -2\epsilon_m \sin\phi_m \sec\epsilon_{\mathrm{POA}} & 1 \end{pmatrix} \begin{pmatrix} \psi_{\mathrm{FOV}} \\ \epsilon_{\mathrm{FOV}} \end{pmatrix} \tag{14}$$

Equation 14 represents a rotation by an angle $\alpha = -2\epsilon_m \sin\phi_m \sec\epsilon_{\mathrm{POA}}$. At $\phi_m = -22°$ we find $\alpha \approx \epsilon_m$, which gives rise to a broadening of the vertical extent of the FOV of up to about 25% for an aspect ratio of 10:1, varying linearly with the mirror elevation. This will certainly complicate the retrievals, and must also degrade the information content.

We can re-derive these first-order results with a quite different approach. It is not difficult to show that the combination of a reflection in a mirror with normal $\mathbf{m}$ followed by a reflection in a mirror with normal $\mathbf{m}'$ is equivalent to a rotation $2\ \mathbf{m} \wedge \mathbf{m}'\, \mathbf{m} \cdot \mathbf{m}'$, which represents an angle of twice the angle between the normals, about an axis parallel to the line of intersection of the mirror surfaces. This can be proved by multiplying out the two reflection matrices, and equating the result to a rotation matrix. The effect on the FOV of rotating the scan mirror can be deduced from this theorem, as follows. The resulting FOV can be found from the initial FOV by 'unreflecting' it in the initial mirror position, and then reflecting it in the final mirror position; the theorem shows that this process is equivalent to a rotation. Thus the effect of scan mirror motion is just to rotate the FOV about an axis, and no distortion is introduced. In two simple cases the results are obvious:

(i) If the scan mirror rotates in elevation at zero azimuth, then the rotation axis lies in the mirror surface, and is therefore the rotation axis of the theorem, and is also normal to the ILOS, which therefore rotates at twice the angular speed of the mirror.

(i) If the scan mirror rotates in azimuth at zero elevation then the same conditions apply, and again the ILOS rotates at twice the angular speed of the mirror.

However in the general case of an elevation scan at constant azimuth, while the rotation axis is still in the plane of the mirror, it is not normal to the ILOS. For small angles ϵ_m the rotation of the FOV can be decomposed vectorially as follows:

$$\begin{aligned} 2\epsilon_m\ (\cos\phi_m \hat{\mathbf{y}} - \sin\phi_m \hat{\mathbf{x}})\ &= 2\epsilon_m \cos\phi_m\ (\cos 2\phi_m \hat{\mathbf{y}} - \sin 2\phi_m \hat{\mathbf{x}}) \\ &+\ 2\epsilon_m \sin\phi_m \tan\epsilon_{\mathrm{ILOS}}\ \hat{\mathbf{z}} \\ &-\ 2\epsilon_m \sin\phi_m \sec\epsilon_{\mathrm{ILOS}}\ \mathbf{n}_{\mathrm{ILOS}}. \end{aligned} \tag{15}$$

The first term in (15) gives the scan in elevation, rotating the ILOS about a horizontal axis normal to the ILOS, through an angle reduced by the factor $\cos\phi_m$, as in (13a). The second term is a rotation about a vertical axis, giving the azimuthal slant to the elevation scan through an angle $2\epsilon_m \sin\phi_m \tan\epsilon_{\mathrm{ILOS}}$, as in (13b). The final term is a rotation about the ILOS itself, rotating the FOV through an angle $-2\epsilon_m \sin\phi_m \sec\epsilon_{\mathrm{ILOS}}$, as in (14).

Finally we use our approximate results to determine the derived azimuth accuracy requirements on ϕ_m and ϕ_{FOV} from LOS elevation accuracy requirements. Equation (10a) gives us the sensitivity of the LOS elevation of a specified detector to scan mirror and FOV angles. Since ϕ_m and ϕ_{FOV} enter symmetrically:

$$\frac{\partial\epsilon_{\mathrm{LOS}}}{\partial\phi_m} = \frac{\partial\epsilon_{\mathrm{LOS}}}{\partial\phi_{\mathrm{FOV}}} = -2\epsilon_m \sin(\phi_m + \phi_{\mathrm{FOV}})\ .$$

The largest magnitude of this occurs at extreme elevations and azimuths and is about 1/40. Thus if we consider an error budget allowance of 0.1 arc sec for elevation uncertainty from this source, the corresponding requirement for knowledge of ϕ_m and ϕ_{FOV} is 4 arc sec. This number needs to be taken together with the requirements on the orthogonality of the mirror axes, and co-alignment of the mirror and spacecraft axes.